Adaptive Smooth Scattered-data Approximation for Terrain Modeling

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Contents

- Scattered-data Approximation
- Overview of Algorithm
- Local Fitting
- Merging Patches
- Results

Scattered-data Approximation

- highly non-uniform scattered data
- continuous approximations
  - error control
  - local refinement
- applications
  - visualization
  - compression / progressive transmission
  - surface reconstruction

Adaptive Approximation
Related Work

- radial basis functions
- splines
  - on triangles (box-, triangular splines)
  - on regular grids (B-splines)
  - on adaptive grids (hierarchical B-splines)
- least-squares fitting
  - large systems of equations
  - feasible for data on regular grids

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Overview of Algorithm

- adaptive clustering (quadtree)
- local fitting with polynomial patches
- merging patches to B-spline surface
- error estimation
- recursive refinement

Adaptive Clustering

- scattered data \((p_i, f_i)\)
- uniform partitioning \(C_0\)
- for \(j=0,1,...,n\)
  - continuous approximation \(F_j(s,t)\)
  - errors \(\Delta f_i = f_i - F_j(p_i)\)
  - refine clusters \(C_j\) with \(\max \Delta f_i > \varepsilon\)
  - replace \(f_i := \Delta f_i\)
- adaptive representation \(\Sigma_j F_j(s,t)\)
Local Fitting

- polynomial patches
  \[ P(s,t) = \sum_{j=1}^{n} c_j \phi_j(s,t) \]
  (\( \phi_j \) bivariate polynomials, deg=1,2)
- least-squares fitting
  \[ A^T A c = A^T f \]
  \[ a_{ij} = \phi_j(p_i) \]
- complexity \( O(m n^2 + n^3) \)
  \( m \) points, \( n = 4, 9 \)
Merging Patches

- discontinuous B-spline representation with multiple knots (Bézier patches)
- reduce multiplicity by knot removal
  - smooth global representation ($C^{\text{deg}-1}$)
  - support increases
  - add clusters at boundary of support

Bilinear Case

- average control points at cluster corners (except for data boundaries)

Biquadratic Case

- remove control points at cluster boundaries (except for data boundaries)
Efficient Evaluation

- level of detail $m$: $F(s,t) = \sum_{j=0}^{m} F_j(s,t)$
- for fixed level $m$:
  - match resolution by inserting knots on coarser levels $F_0 ... F_{m-1}$
  - add corresponding control points of $F_0 ... F_m$
  - evaluate only one representation, $F := F_m$

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Results

- data sets courtesy of USGS,
  - Crater Lake 18 800 points (11.8 %)
  - Seattle 587 000 points (0.33 %)
- downsampled, based on local curvature
- 6 levels computed, form $32 \times 24 = 786$ to $1024 \times 786 = 786 400$ clusters
Results

• Crater Lake
  0.26% L^2-error (threshold $\varepsilon = 0.5\%$)
  20,600 clusters on all levels
    (2.6% of 786,400)
  4.3 sec (3.3 for bilinear)

• Seattle
  0.79% L^2-error (threshold $\varepsilon = 1\%$)
  319,400 clusters on all levels (40.6%)
  23.1 sec (11.8 for bilinear)

Crater Lake

18,800 points

768 clusters, 4.65% L^2 ... 20,600 clusters, 0.26% L^2

Seattle

587,000 points

3,072 clusters, 13.3% L^2 ... 319,400 clusters, 0.79% L^2
Conclusions and Future Work

• selected scattered data provide basis for efficient continuous approximation
• problems in sparse regions near steep gradients
• feasible extensions
  – 3D domain (volumes)
  – 3D range (vector fields, FFD's)